AAS 93-623

MARS GRAVITY FIELD FROM DUAL SATELLITE OBSERVATIONS

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Two strategies are examined in this paper to improve the spherical harmonic representations of the Martian gravitational field. Continuous observations over a week-long data arc and a 30-th degree gravitational field are assumed for analysis. In the first strategy, it is proposed to obtain Doppler data from the Mars Observer spacecraft at the two Russian tracking stations at Ussuriysk and Yevpatoriya and at the German station at Weilheim, in addition to the Deep Space Network (DSN) stations at Goldstone, Canberra and Madrid. In the second strategy, satellite-to-satellite (S1'S) Doppler tracking data is exclusively obtained on the relative velocity between two satellites in orbit around Mars in the high-low configuration. For analysis, the Mars Observer spacecraft in the low-orbit is paired with the high-altitude Navigation or Communications Relay Satellite(s) of anticipated future missions. RMS errors in the power spectra of the gravitational field are reduced by about a factor of 2 with tracking data from 6 stations as in the first strategy (instead of only the 3 DSN stations) and by nearly a factor of 5 for the second strategy with exclusively STS tracking data. Meter-level geoidal uncertainties resulting from observations with only DSN stations, are reduced to about 25 cm with 6 stations participating in tracking the Mars Observer satellite. With the superior performance of STS tracking, geoidal errors at 5 centimeters or below can be obtained.

INTRODUCTION

An accurate knowledge of the Martian gravitational field is very essential especially for the navigation of low-altitude reconnaissance satellites, which will play a vital role in our continued efforts for the exploration of Mars, Smith[1] *et al.*, and Esposito[2,3] provide an excellent introduction to the current status of the knowledge of the Martian gravitational field and the need for further improvements. Scientific data can be more accurately interpreted with improved navigational accuracy for the observational satellites,

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Geological and geophysical studies on the internal structure of Mars as well as the internal distribution of density anomalies due to variations in temperature and composition, will be significantly facilitated if the planetary gravitational field can be determined accurately. A precise and dc.tailed description of the gravity field will be very helpful in identifying "frozen mbits" about the planet for deploying low-altitude satellites.

Till recently, an 18-th degree and order spherical harmonic field published by Balmino[4] and co-authors in 1982, provided the highest resolution for the Martian gravity field with a half wavelength of about 600 km. The Balmino field was derived from tracking data obtained from Mariner 9 and the Viking 1 and 2 orbiters. Limitations due to computational resources available at that, time, diminished the accuracy and resolution of the field and hence its usefulness for many applications[1]. Smith[1] and co-authors have just published a 50-th degree spherical harmonic model, called GMM-1 (Goddard Mars Model-1), for the Martian gravitational field, derived from 1100 days of coherent 2-way Doppler data in the S-band (2.2 GHz) obtained by the NASA Deep Space Network (DSN) stations from Mariner 9 and the Viking 1 and 2 orbiters, during the period from 1971 through 1979. Besides the better spatial resolution of about 200-300 km (half wavelength), the GMM-1 gravity model[1] "represents satellite orbits with considerably better accuracy" and "shows greater resolution of identifiable geological structures". Although the GMM-1 model is derived from essentially the same tracking data, the authors have taken advantage of the availability of better computers, improved analytic techniques and more accurate knowledge of the measurement process and the forces on the spacecraft, in obtaining the 50-th degree and order gravitational field.

The Mars Observer mission currently in progress will provide an outstanding opportunity to refine the gravity model even further. During the gravity calibration phase, the Mars Observer spacecraft will be in a low-altitude (378.1 km), near-circular orbit (eccentricity, $e = 0.004$) with a nodal precession rate of 0.524° per day and an orbital period of approximately 117 min. The spacecraft is in a near-polar orbit and the ground-track will almost repeat after every 88 orbits (7 sol repeat cycle). With the cent inuous '2- way coherent X-band tracking data (both Doppler and range) to be obtained by all three DSN stations at Goldstone, Canberra and Madrid, the accuracy of the gravity field will be improved tremendously with the GMM-1 model providing the *a priori* values,

in this context, two different. strategies to further improve the gravitational field have been examined and the preliminary results of the analysis are presented in this paper. The first strategy calls for continuously tracking (with Doppler data) the Mars Observer spacecraft for a period of 1 to 3 weeks, not only from the three DSN stations, but also from the two Russian deep space network stations at Ussuriysk and Yevpatoriya and the German tracking station at Weilheim. The total "saturation" coverage provided by the 6 tracking stations contributes to significant improvement in determining the gravity field.

in the second strategy, satellite-to-satellite (STS) Doppler tracking data is obtained on the relative velocity between two satellites in orbit around Mars in the 'high-low' configuration and the gravity field is recovered. The Mars Observer spacecraft serves as the satellite in the low-altitude (380-430 km) orbit. The Navigation Satellite (N AVSAT) discussed in Ref. 5, in a near-circular orbit (nominally, $e = 0$), with a semi-major axis of 16860 km and at an inclination of 50° (near-frozen orbit) serves as the '(high satellite". Since the realtive velocity between the high and low satellites is derived essentially from the high frequency gravitational harmonics, extremely interesting results are obtained in this case. The Con~-

munication Relay Satellites discussed in Ref. 6, will also be quite acceptable candidates for the high-altitude satellite(s).

GRAVITY FIELD DETERMINATION

From standard texts[7,8] and references[1,2], it may be seen that the Doppler signature from a spacecraft in orbit around Mars contains information on the spherical harmonic expansion coefficients for the planetary gravitational field. Hence determining the gravity field simply reduces to a problem of parameter estimation in spacecraft navigation. Software for orbit determination and covariance studies form the primary tools for analysis discussed in this paper. For the sake of brevity, the analytic details in the above references are not repeated here.

Both strategies outlined in the introduction for determining the Martian gravitational field with greater accuracy, were evaluated by covariance studies with computer simulations. The LEXUS[9] software was used in both cases for consistency, since it more readily accommodated multiple spacecraft and satellite-to-satellite (S'1'S) tracking data.

Several cases have been considered for the "saturation coverage" of the. Mars Observer (M O) spacecraft being tracked by the six deep space network stations with international collaboration. The simulations were carried out with 2-way and 3-way Doppler tracking data obtained continuously for a week on 4 different occasions when the MO orbital plane was at an inclination of 24°, 44°, 57° or 89° to the plane-of-sky (1'0S). For instance, the approximately 'face-on' view-geometry of 24° between the MO orbital plane and the plane-of-sky is expected to occur around April 23, 1995. The near 'edge-on' view-geometry with an inclination of 89° is anticipated about May 15, 1994. The 1-week period from March 15, 1996 corresponds to the case with an orbit inclination (to }'0S) of 44°. During the week of October 25, 1995, it is anticipated that MO orbital plane will be inclined at 57° to the plane-of-sky. Apart from the view-geometry just discussed, the station rise-set periods for observation from the tracking stations and the occultation of the spacecraft by Mars also differed from case to case. Due to these reasons, nearly 59000 data points (simulated) were available for the l-week period in May 94, whereas 97,000 data points were processed in the simulation for the 1-week period in April 95. All these details have been taken into account in the computer simulations.

In general, a one-week data arc was investigated in each case, consistent with the 7-sol repeat cycle of the Mars Observer satellite. However, the information matrices pertaining to the cases for May 94, April 95 and March 96 have also been combined to examine the advantages to be derived from 2-week and 3-week long composite data arcs. The details for such cases will be specifically described in each instance when the pertinent results are discussed.

As the computer simulations were carried out with the 6 deep space network stations tracking the Mars Observer, in most instances the gravitational field was also evaluated with radiometric data only from the three NASA Deep Space Network (DSN) stations or from the 3 DSN stations and the two Russian stations only. The results for these various cases are discussed at the end specifically bringing out the advantages of including the Russian and German stations for observation.

As for the second strategy with STS tracking data, only one case has been studied, The nominal trajectory of the Mars Observer spacecraft from April 23, 1995 was considered as the low-altitude satellite ephemeris for the entire data span of 1 to 3 weeks. The Navigation Satellite orbital parameters are given at the end in Table 1. The Navigation Satellite in the high-altitude orbit (with a semi-major axis of 16860 km) is much less sensitive to the high frequency gravitational harmonics. Approximate analysis[10] had confirmed the advantages in recovering the high frequency gravitational field from the relative velocity between two satellites in the "high-low" configuration. There was no attempt made to optimize the phasing between the high and low satellites,

ESTIMATION PROBLEM

At the outset it was decided to restrict the scope of the analysis only to evaluate the relative merits or advantages of the several cases and strategies. For a prelim i nary effort, t he total number of parameters was limited to a thousand (or even lower) in order to minimize the demand on computer resources and to examine whether some reasonable conclusions can be drawn in a relatively short time.

Hence the Martian gravitational field was limited to a truncated 30-th degree and order spherical harmonic expansion; it was derived from a preliminary version of the GMM-1 model, known also as the MGM-635 model, by truncating the 50-th degree (and order) field to the 30-th degree. The zonal and tesseral harmonic coefficients for the 30-th degree and order gravitational field work out to 957 unknowns for the parameter estimation problem. The GM-value (the planetary gravitational constant) for Mars was estimated also.

For Mars Observer, the spat.cc.raft state and drag-coefficient were estimated also. in all, for the navigation problem of simply tracking the MO from the 6 deep space network stations, there were 965 parameters to be estimated, comprised of the 957 gravitational harmonic coefficients, the GM-value for Mars, the position and velocity of the spacecraft (6 parameters) and the drag- coefficient for the. satellite in the low-altitude orbit. It must be remarked that no other error sources were taken into account in the analysis by computer simulations. 'l'he major error sources not considered for analysis are the station location errors, clock errors, ionospheric and tropospheric corrections to the observations, uncertainties in the solar pressure model, gas leaks or random accelerations and errors in the earth-Mars ephemerides. It is fully understood and conceded that the final error statistics on the gravitational field obtained in this analysis will be far too optimistic; however, relative comparisons of the different cases and strategies are expected to be substantially correct.

When satellite-to-satellite (STS) tracking data is obtained as for the second strategy, the position and velocity of the NAVSAT in the high-altitude orbit were added to the list of estimated parameters, contributing to a total of 971 unknowns. It is anticipated that the NAVSAT or any other corresponding high-altitude satellite (such as the Communication Relay Satellites) will carry an ultra-stable oscillator of superior characteristics and computers sufficiently fast that it will generate the. navigation signals to be transmitted to the Mars Observer and process the returned signals on-board the satellite. The instantaneous differenced-Doppler data from a deep space network station on the earth to the two satellites in orbit around Mars will be an alternate implementation of the same strategy. However, this aspect will not be discussed any further in this paper.

The square-root filter option has been chosen for analysis in the LEXUS navigation software. Also, it is assumed that *a priori* no information is available on the harmonic gravitational coefficients.

DATA SCHEDULING AND DATA NOISE

In order to estimate the accuracy with which the 30-th degree and order gravitational field can be recovered in the different cases, Doppler data from the Mars Observer spacecraft was scheduled at 20 second intervals continuously during the week(s) -long data arc, whenever the spacecraft was visible at the deep space network station or at the Navigation Satellite as appropriate. Corresponding to the half wavelength of about 355 km for the 30-th degree field, the sampling interval of 20 seconds ensured that there will be nearly 5 data points in each spatial half wave, with the spacecraft traveling at about 3.4 km/s.

It should be remarked that only the NASA Deep Space Network stations at Goldstone, Canberra and Madrid can transmit navigation signals to the Mars Observer spacecraft and receive also. The tracking stations at Ussuriysk, Yevpatoriya and Weilheim can operate only in the receiving mode and hence they actually participate only in the "3-way Doppler data" acquisition mode, However, since the NASA tracking stations continuously cover the spacecraft over each 24-hour cycle without any break (except for occultations), the view-periods of all stations are fully covered with 2-way and/or 3-way Doppler data.

Since the analysis is considered only preliminary, the minimum elevation angle for cutting off data was taken to be 4° to ensure continuous coverage across the tracking stations. The low cut-off angle provides approximately similar conditions for comparison, without any tremendous effort to modify the data arc in each case to accommodate changes which are not considered significant for analysis.

Most of the discussions in this paper will be restricted to 2-way and 3-way Doppler data only, when deep space network tracking stations are involved. All Doppler data, whether 2-way or 3-way, have been weighted at 0.1 mm/s for estimation.

So far, it has not been mentioned that same-beam interferometric[5] (SBI) data was also considered in the analysis, Any discussion of SBI data was so far avoided, essentially to focus attention on the more significant contribution from Doppler data in the recovery of the Martian gravitational field. However, in most cases, whenever there was an overlap of tracking stations for the chosen baselines, same-beam interferometric (SBI) data was also obtained and considered for analysis. The Goldstone-Canberra, Goldstone-Madrid, Madrid-Ussuriysk, Goldstone-Ussuriysk and the Madrid-Yevpatoriya baselines were taken into consideration. Throughout every overlap of the two tracking stations on each baseline, SBI data was obtained at 20 second intervals and the data was weighted[11,12] at 0,001 mm/s. It will be remarked that same-beam interferometric data did not appear to improve the results for gravitational field in any significant manner.

When examining the second strategy, satellite-to-satellite Doppler data was obtained at 20 second intervals (for the same reasons as earlier outlined), whenever the two satellites could communicate to each other without being occulted. Since the two spacecraft are not likely to be more than 21,000 km apart, in the absence of interference due to solar plasma and media errors as for data received at the tracking stations on the earth, the data weight for satellite-to-satellite Doppler data was taken to be 0.01 mm/s. Essentially continuous coverage was assumed for the entire data arc of 1 to 3 weeks, except for outages due to occultation. Doppler data from the two satellites were obtained at the DSN stations at 10 minute intervals, to impose some realistic constraints on the spacecraft states for the estimation problem.

The X-band transponder on the Mars Observer spacecraft receives navigation signals at 7.16 GHz and returns them back to the earth at 8.42 GHz. Hence the weights attributed to the different types of data in the paragraphs above, may be considered quite realistic.

CRITERIA FOR RELATIVE MERIT EVALUATION

The relative merits of the different cases and strategies examined here to determine the Martian gravitational field, can be evaluated by various measures. For instance, correlating the surface features or topography to the gravity field as well as predicting the trajectory of a spacecraft in orbit around the planet are two (by now almost standard) measures which have been considered in References 1 and 4. In this paper, however, the relative merits of the several cases are discussed by presenting comparisons of the 1 -σ values of the geoidal uncertainties (in meters) plotted as contours over the Martian latitude-longitude landscape. Also, the root-mean-square (RMS) errors (or uncertainties) in the the gravitational power spectra are comparatively examined for dependence on data type., data arc. length, view-geometry and station coverage.

GEOIDAL UNCERTAINTIES

The ground-tracks of the Mars Observer spacecraft (or more precisely the sub-satellite points on Mars) at the data points when 2-way Doppler data only are obtained just at the DSN stations, are plotted in Figure 1, for the week-long data arc from the 23rd to the 30th of October, 1995. The corresponding results on geoidal errors (1-u values) expressed in meters, are shown in Figure 2, as contour plots on the latitude-longitude plane. Some simple observations are readily made, Due to occultation, the coverage is relatively sparse in the equatorial region in comparison to the regions at high latitudes. Correspondingly areas of maximum geoidal uncertainties of the order of 65 cm may be observed at 150° W longitude at about 0°- 10°N latitude. in fact, the two gaps in the coverage west of 120°W longitude in Figure 1, are almost precisely reflected in the large geoidal uncertainties in the contour map in Figure 2. The coverage is dense at northern latitudes above 30" N, but only at latitudes below 50°S. Correspondingly the line at latitude 40°S indicates a geoidal uncertainty value in excess of 35 cm, while at the 40° N latitude line, the geoidal errors remain close to about 32 cm. It is not to be misconstrued that gaps in coverage will always be attendant with large values in geoidal errors. For instance, the 50-55 cm peaks in geoidal errors near the equatorial region at 60°E and 100°E longitude (Fig. 2), are not matched by any gap(s) in the coverage seen in Figure 1.

For the same week in October 1995, the "ground-track" coverage at data points with all 6 deep space network stations tracking the Mars Observer satellite with 2-way and 3-way Doppler data is shown in Figure 3 and the corresponding contour plots of the geoidal erros (1 -u values in meters) are presented in Figure 4, It may be observed that the coverage is more uniform and dense in Figure 3 in comparison to Figure 1. Correspondingly, the maximum geoidal uncertainties are at about 40 cm as against 65 cm for the earlier case, as seen from Figures 4 and 2 respectively. Furthermore, the gradual or gentle sloping of the contours in Figure 4 in contrast to the steep peaks in Figure 2, is indicative of the advantage derived from superior coverage obtained with 6 stations tracking the satellite as against only the three DSN stations providing the coverage in Figure 1. It is of interest to note that 54,835 sub-satellite points are mapped onto Figure 3, in contrast to the 24,548 points in Figure 1.

The geoidal uncertainties obtained with 1 week of satellite-to-satellite Doppler tracking between the Navigation Satellite and the Mars Observer spacecraft are shown plotted in Figure 5. It is very interesting to note that the maximum 1-σ value of the geoidal error is less than 5 cm and at the high latitudes, it is at or below 2 cm. In this case, at the high southern latitudes, the geoidal error is smaller than at the corresponding high northern latitudes and this is essentially due to the accidental phasing of the initial conditions of the two satellites. It is also interesting to note that a total of about 23,333 STS data points were obtained over the one-week period of observation, in contrast to the 54,835 points from the 6 deep space network stations on the earth over the 1 week period in October 95 just discussed earlier. The result is clearly indicative of the superiority of satellite-to-satellite observations in determining the high-frequency gravitational field.

Besides these three cases just discussed above in detail, several others have also been examined for geoidal errors. In general, the various cases pertain to the 4 different l-week periods of observation from the tracking stations On the earth (such as May 94; April 95; October 95; March 96). Hence each is associated with a specific plane-of-sky view-geometry as already discussed in the section on "gravity field determination". The various cases are further characterized by the data types included for analysis; such as "DSN 2-way data only" or "DSN 2-way and 3-way data", where the acronym "DSN" strictly refers to the NASA Deep Space Network stations at Goldstone, Canberra and Madrid (just as elsewhere throughout this paper). The third data type category consists of 2-way and 3-way Doppler data obtained at all the 6 tracking stations (including the two Russian stations and the German station in addition to the DSN). Therefore, there are twelve different cases, each uniquely associated with a specific period of observation and types of data considered for analysis.

Besides these twelve cases, the information matrices of the week-long observations during May 94 and March 96 were combined to examine the results from the composite 2-week data arc, for DSN 2-way and 3-way data as well as for 2-way and 3-way data obtained at all 6 tracking stations. Similarly, a composite data-arc for 3 weeks including the data from May 94, March 96 and April 95 has been analysed with the participation of all 6 tracking stations (2-way and 3-way Doppler data included).

As for satellite-to-satellite (STS) tracking, data spans of 1, 2 and 3 weeks have been examined. The view-period and view-geometry associated with earth-based observations are not relevent in this case. The phasing between the two satellites is important, but its influence has not been studied. The Mars Observer satellite was tracked from April 23, 1995 continuously for a period of 3 weeks with Doppler data from the Navigation Satellite with the initial conditions given in Table 1,

Results on geoidal uncertainties for the various cases described above are presented in Table 2 at the end. The actual entries for the errors are 1-σ values, but averaged over the whole latitude-longitude plane evaluated at grid points at 2° intervals.

It is easily seen that tracking from 6 stations with 2-way and 3-way Doppler data results in an improvement of about 20%, 26%, 29% and 31'% over tracking from only the 3 DSN stations with the same data types for l-week data arcs during October 95, March 96, May 94 and the composite 2-week data arc comprised of the May 94 and March 96 observations, Hence it is felt that tracking from 6 stations must be preferred for determining the high frequency Martian gravitational field.

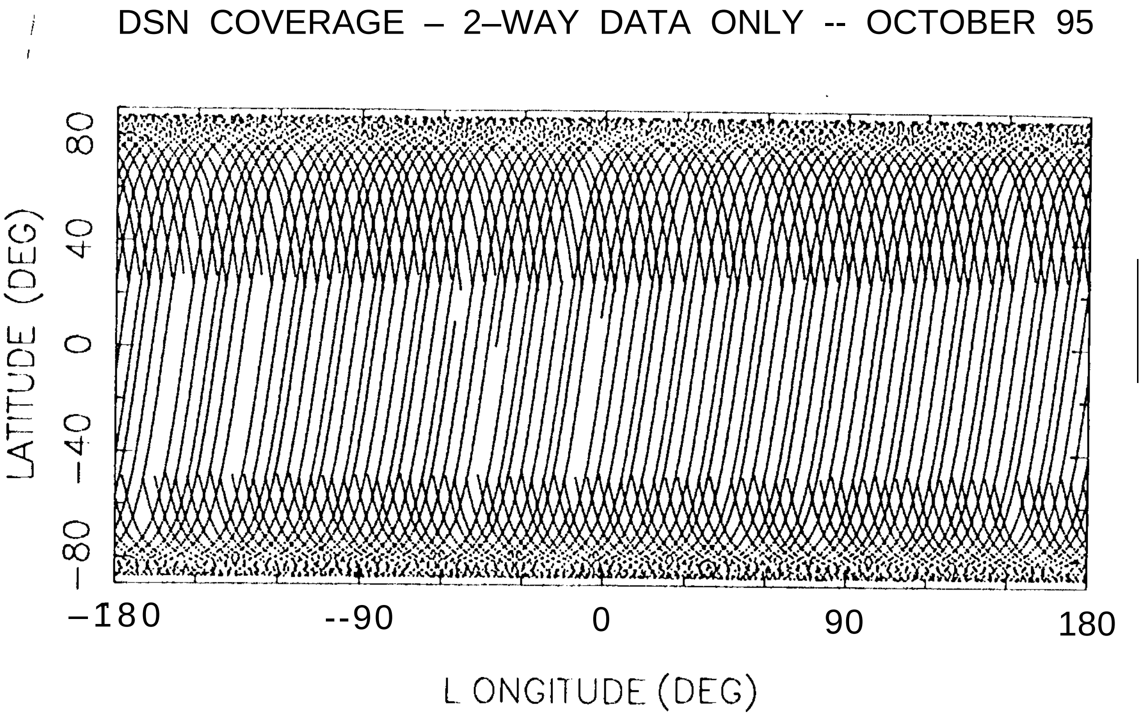


Figure 1.

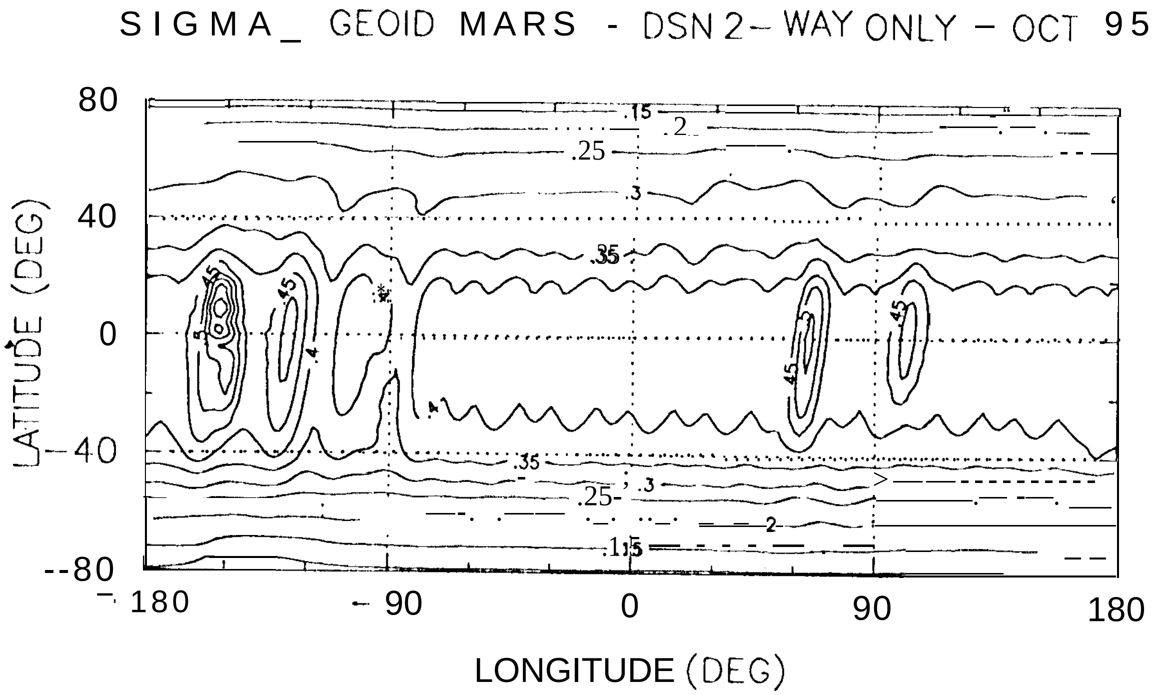


Figure 2.

(Contour values of geoidal errors are in meters.)

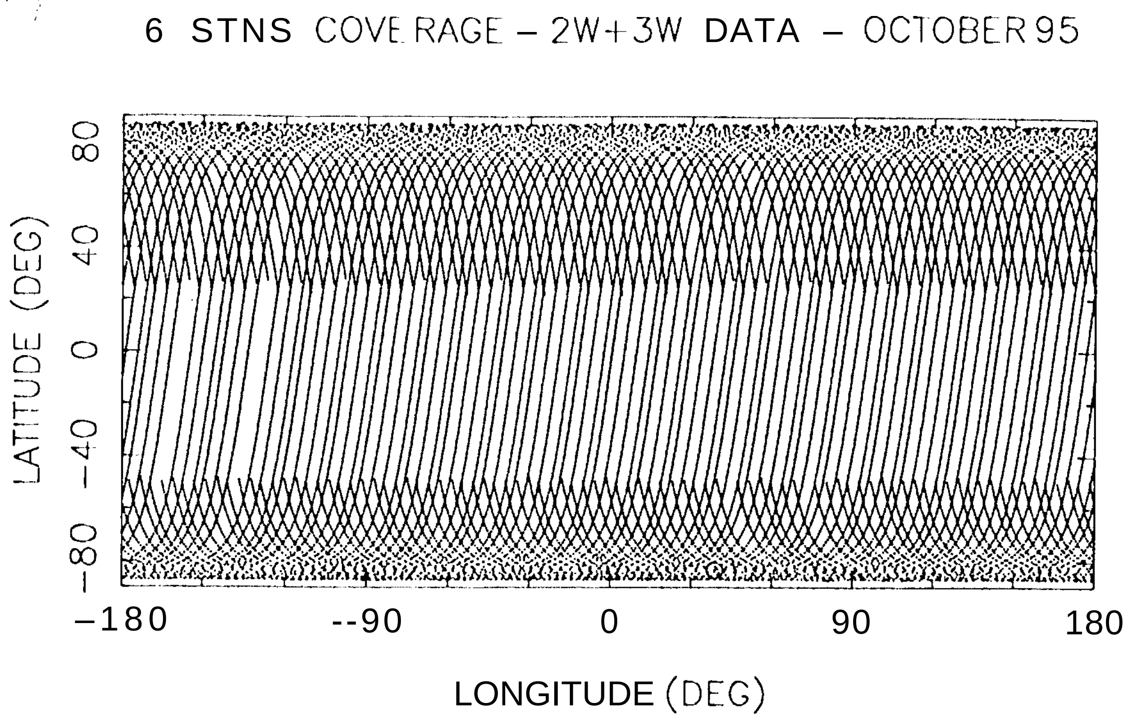


Figure 3.

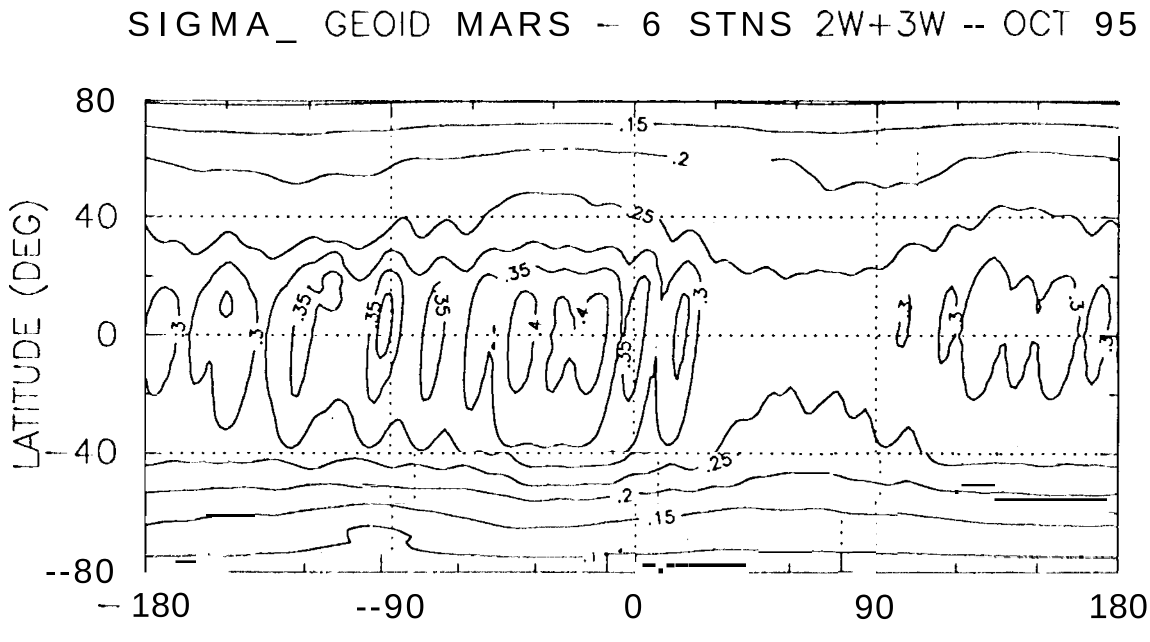


Figure 4.

(Contour values of geoidal errors are in meters.)

SIGMA_GEOID MARS - STS DATA * 0.01 MM/S * - APR 95

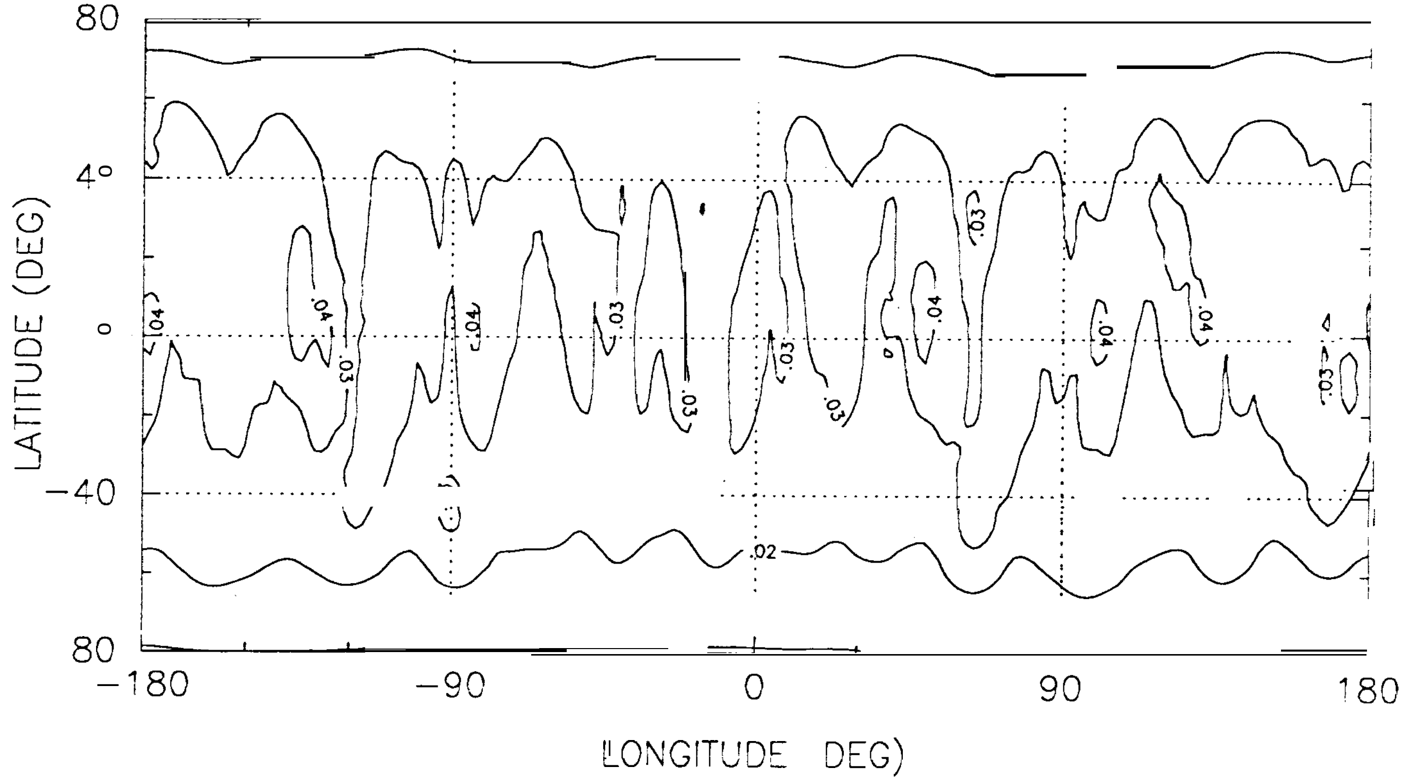


Figure 5.

(Contour values of geoidal errors are in meters.)

It is also seen that the near "face-on" geometry during the week of April 23, 1995 with 96,816 data points (from 6 stations) results in an average geoidal error of 16 cm, while the corresponding value is 15 cm for observations during May 94, with the "edge-on" view-geometry, but with only 58,835 data points. In fact, except for the composite data-arcs, the edge-on view-geometry results in the lowest average geoidal error as seen from the entries in the 3rd row of Table **2.** This implies that the edge-on geometry clearly provides superior information in the observations compared to others.

The composite data-arcs indicate lower values of average geoidal errors, as seen by the entry of 8 cm for the 3-week data arc. However, it is likely that 3 weeks of observations centered about May 15, 94 would have resulted in a lower geoidal error value than the 8 cm for the composite 3 week data-arc.

It is interesting to note that the addition of thousands of Same Beam Interferometric (SBI) data points from 5 different baselines earlier described, did not contribute to any further reduction in the geoidal errors from the values already obtained with just 2-way and 3-way Doppler data at the 6 tracking stations during the same view-periods (April 95 and May 94). It was verified that errors in the spacecraft state were, however, reduced with the addition of SBI data. Some sensitivity studies have been carried out with different data weights, but so far the results indicate that interferometric data do not contribute significantly to the determination of the gravity field.

It may be mentioned that many of these cases have been examined with the Weilheim tracking station excluded from observations. However, the results are substantially unaltered from their values for geoidal errors as seen in the entries in the third row of Table 2, corresponding to observations with six tracking stations.

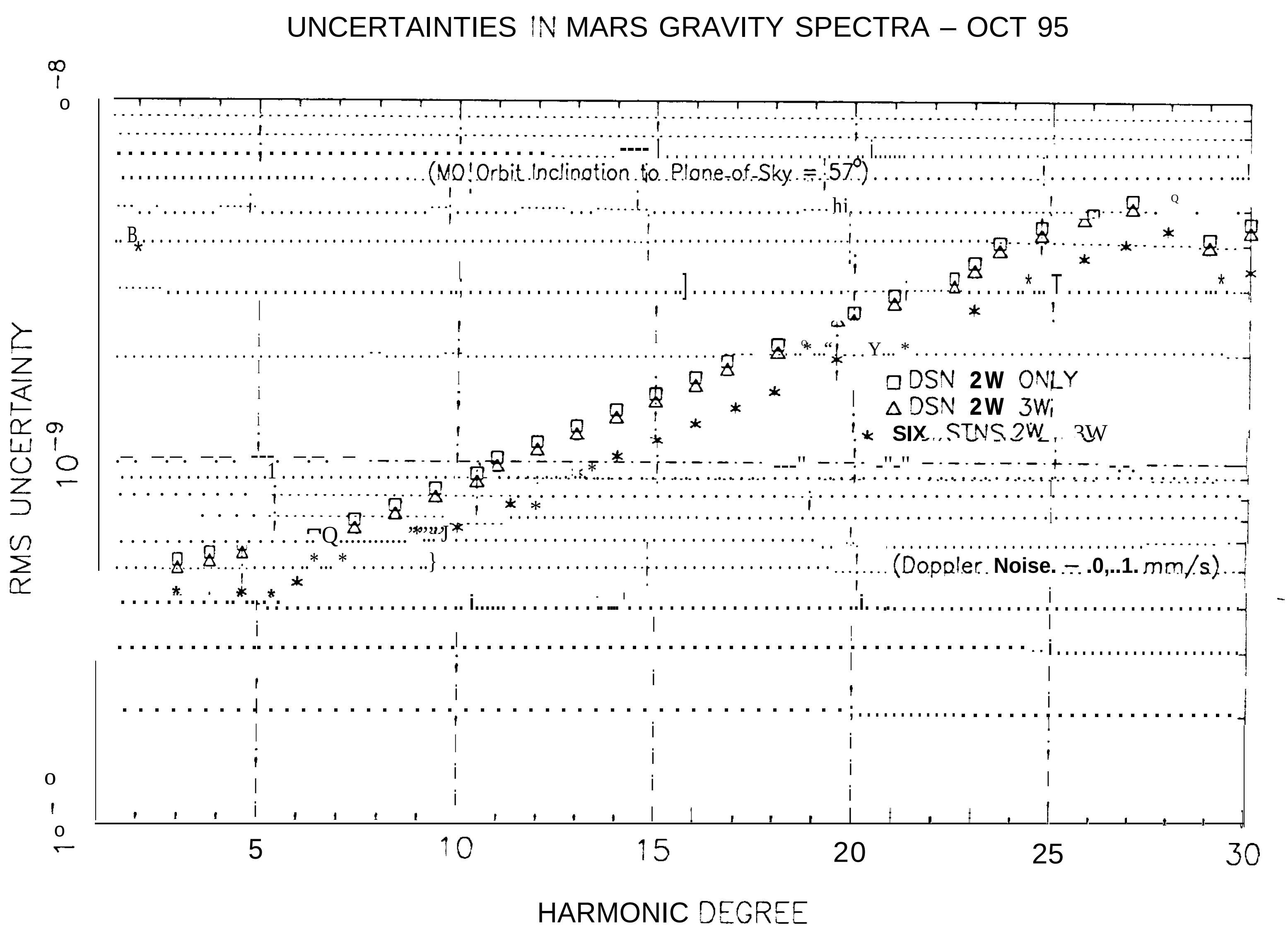
The average geoidal uncertainties obtained with satellite-to-satellite (STS) tracking data are given in the last row of Table 2. It is extremely interesting to note that the geoidal errors may be limited to a few centimeters at the worst and in particular, such remarkable results are obtained with as few as 23,000 data points in comparison to the 59,000 and 97,oOO observations necessary with 6 stations on the earth tracking the Mars Observer satellite. It must be remarked that when STS data is considered for analysis in this paper, the gravitational field is recovered exclusively from STS data only.

RMS ERRORS IN MARS GRAVITY SPECTRA

In addition to the geoidal uncertainties resulting from errors in the spherical harmonic expansion coefficients of the Martian gravitational field, the root-mean-square errors in the power spectra of the gravity field have been determined to evaluate the relative merits of the two strategies under consideration. For purposes used in this paper, the RMS error in the Mars gravity spectra is given[4] by

$$\sigma_n(\epsilon) = \left\{ \frac{1}{(2n+1)} \left(\sum_{m=0}^{n} \left[\sigma^2(C_{nm}) + \sigma^2(S_{nm}) \right] \right) \right\}^{\frac{1}{2}} \quad \textbf{(1)}$$

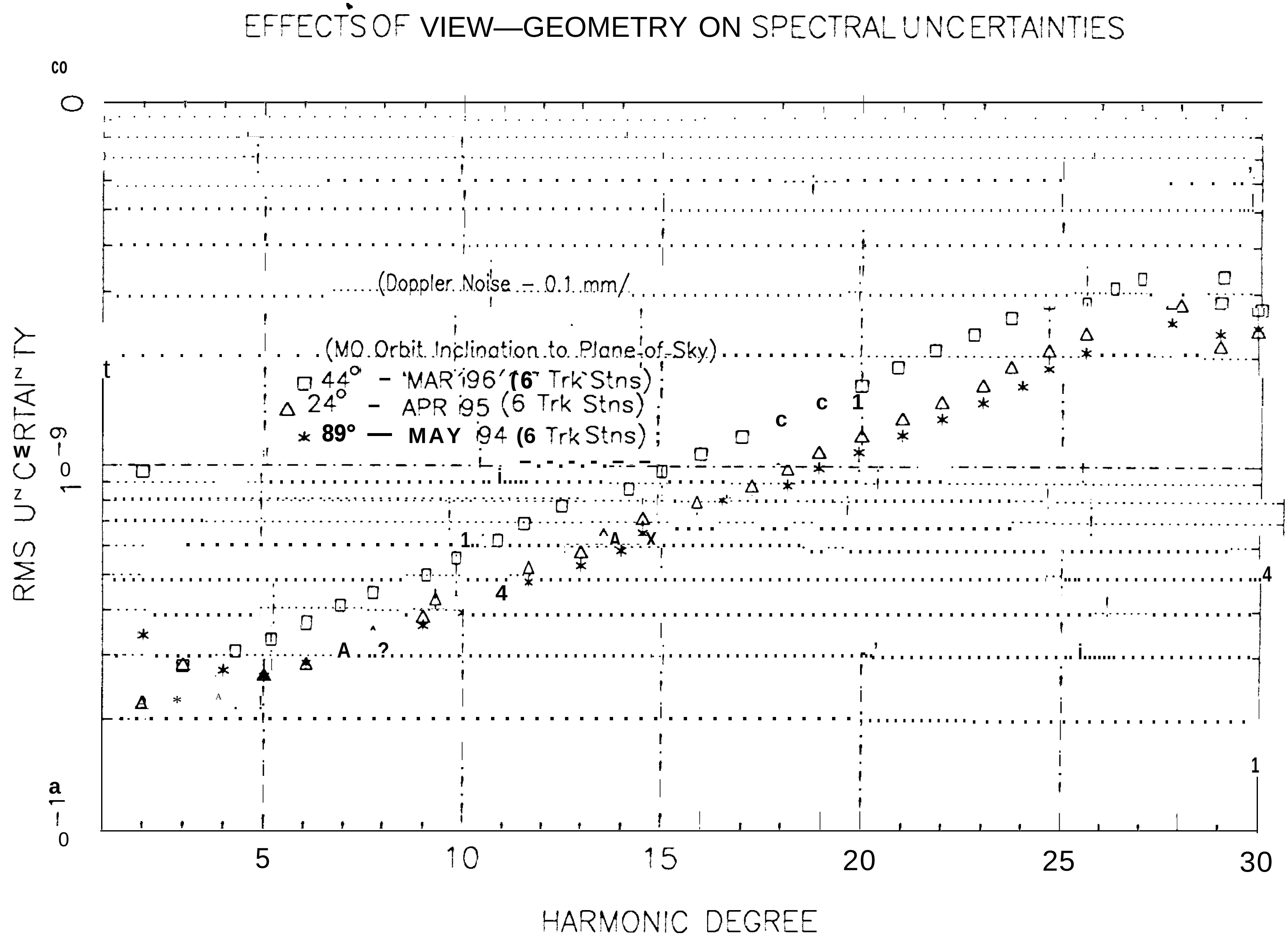
corresponding to the n-th harmonic degree. C_{nm} and S_{nm} in the above equation are the cosine and sine series spherical harmonic expansion coefficients of the Mars gravity potential corresponding to the n-th degree and of order m. $\sigma^2(C_{nm})$ denotes the variance in C_{nm} and correspondingly, $\sigma^2(S_{nm})$ represents the variance in S_{nm}.



UNCERTAINTIES IN MARS GRAVITY SPECTRA – OCT 95
(MO Orbit Inclination to Plane-of-Sky = 57°)
DSN 2W ONLY
DSN 2W 3W
SIX STNS 2W 3W
(Doppler Noise = .0.1 mm/s)
RMS UNCERTAINTY
HARMONIC DEGREE
5
10
15
20
25
30

Figure 6.

2



EFFECTS OF VIEW—GEOMETRY ON SPECTRAL UNCERTAINTIES
(Doppler Noise = 0.1 mm/
(MO Orbit Inclination to Plane-of-Sky)
44° - MAR 96 (6 Trk Stns)
24° - APR 95 (6 Trk Stns)
89° — MAY 94 (6 Trk Stns)
RMS UNCERTAINTY
5
10
15
20
25
30
HARMONIC DEGREE

Figure 7.

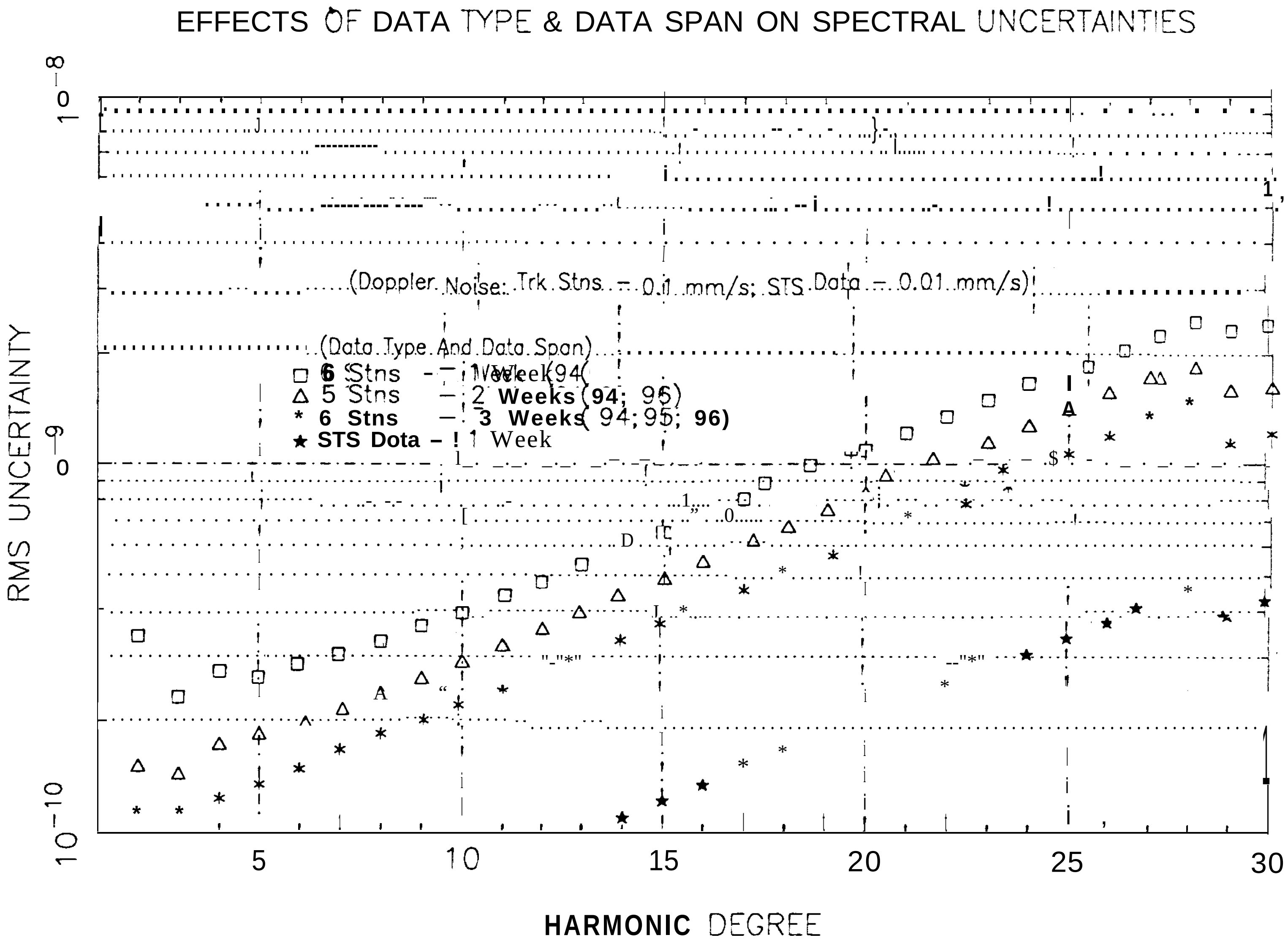

EFFECTS OF DATA TYPE & DATA SPAN ON SPECTRAL UNCERTAINTIES
(Doppler Noise: Trk Stns = 0.1 mm/s; STS Data = 0.01 mm/s)
(Data Type And Data Span)
6 Stns — 1 Week (94)
5 Stns — 2 Weeks (94; 95)
6 Stns — 3 Weeks (94; 95; 96)
STS Data — 1 Week
RMS UNCERTAINTY
10^{-8}
10^{-9}
10^{-10}
5
10
15
20
25
30
HARMONIC DEGREE

Figure 8.

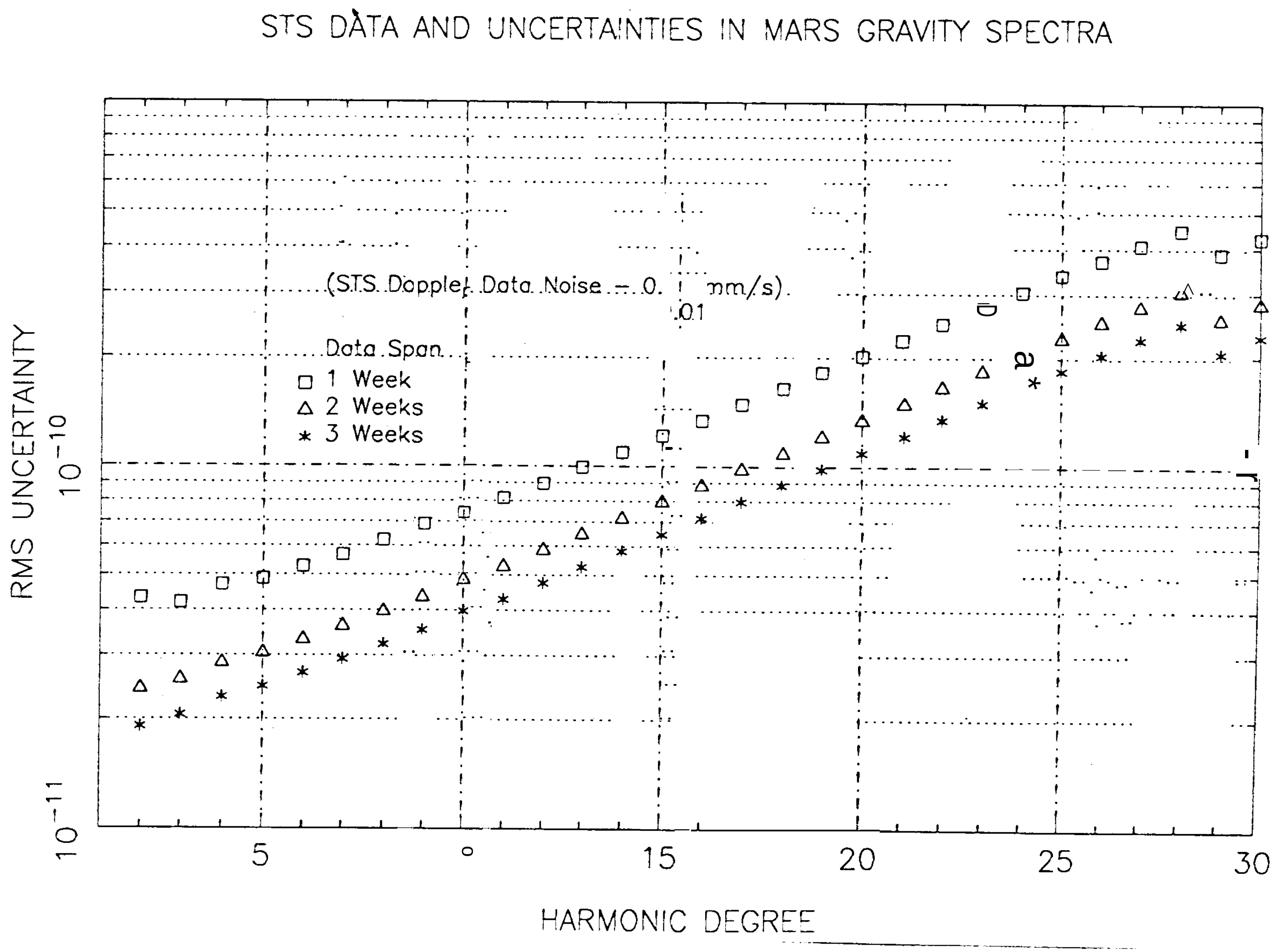

STS DATA AND UNCERTAINTIES IN MARS GRAVITY SPECTRA
(STS Doppler Data Noise = 0.01 mm/s)
Data Span
1 Week
2 Weeks
3 Weeks
RMS UNCERTAINTY
10^-10
10^-11
5
15
20
25
30
HARMONIC DEGREE

Figure 9.

This definition is consistent with equations (2) through (4) in Ref. 4. $\sigma_n(\epsilon)$ denotes the root-mean-square error for the n-th degree, with ϵ introduced to denote error spectral analysis, In Figures 6 through 9, the RMS error, $\sigma_n(\epsilon)$ is plotted against the harmonic degree, n for the various cases.

From Figure 6, it is seen that with 6 stations tracking the MO spacecraft, the rms uncertainty corresponding to degree n, is approximately reduced by 50%, when compared to the results from DSN tracking only. This reduction in RMS errors is manifest not only for the case shown in Figure 6 (corresponding to October 25 through November 1, 1995 view-period), but for all the 4 different cases examined.

In Figure i', the RMS error spectra corresponding to different view-periods are compared to examine the influence of view-geometry in determining the gravity field. Just as already discussed in the case of geoidal errors, the "edge-on" view-geometry is seen to give the best results, closely followed by the "face-on" configuration for observation.

Results on RMS error spectra with different data spans are shown in Figure 8, along with the results for the case with STS data for gravity field determination. Except for the curve for STS data, the other 3 curves correspond to observations with 2-way and 3-way Doppler data from all 6 tracking stations. The advantage of longer data-arcs is clearly seen; but the more striking feature is the reduction in error spectra with STS data, by nearly a factor of 5 over what otherwise is obtained with the best coverage provided by deep space network stations on the earth. Finally, from Figure 9, it is seen that a further improvement by a factor of 2 can be obtained by extending the period of observation with STS data to 3 weeks instead of 1.

REMARKS AND CONCLUSION

It was already stated that several major error sources were not taken into consideration in the foregoing analysis, particularly when Doppler data is obtained at the deep space network stations on the earth. Hence it is quite unlikely that the results discussed above will be valid in terms of absolute values; however, the relative merits of the different cases and strategies as seen from the simulation results are considered to be substantially correct. At least for circular orbits, it seems that observations with "edge-on" view-geometry will provide more accurate determination of the gravity field. Saturation coverage from 6 stations is preferable to data obtained at a few stations only. Satellite-to-satellite Doppler tracking is, indeed, significantly superior to earth-based observations for determining the Mars gravity field. Although in the analysis, the gravity field was truncated to the 30th degree, the results are likely to follow the same trend for higher degree and order representations also.

ACKNOWLEDGEMENTS

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration,

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Table 1. Satellite Orbital Parameters

Parameter	Mars Observer	NAVSAT
Semi-major axis - a, km.	3766.16	16860.16
Eccentricity - e	0.004049	0.0 (circular)
Inclination - i, deg	92.87	50.0 (near-frozen orbit)
Node - Ω, deg	165.26	150.0
Arg. of Periapsis - w, deg	-90.0	-90.0
Time at Periapsis - t_p, sec	0.0	0.0
Epoch	23-APR-1995 00:00:00	23-APR.-1995 00:00:00

Table 2. Average Geoidal Uncertainties
(Averaged 1σ Values in meters)

(Data Weight: Doppler data -0.1 mm/s: STS data - 0.01 mm/s; SBI data - 0.001 mm/s)

Epoch	Ott 25, 95	Mar 1.5, 96	Apr 23, 95	May 15, 94		
Data Arc	1 week	1 week	1 week	I week	2weeks	3weeks
MO orbit Incl to POS	57°	44°	24°	89°	(44°, 89°)	(44°, 89°, 24°)
DSN 2-way (2w) Data only	0.32 (24,54s)4	0.32 (22,364)	–	0.25 (19,516)	–	–
DSN 2-way + 3-way (3w) Data	0.30 (31,217)	0.27 (30,643)	–	0.21 (28,388)	0.16	–
6 Tracking Stations* 2W + 3W	0.24 (54,835)	0.20 (60,067)	0.16 (96,s16)	0.15 (5s,s35)	0.11	0.08
6 Tracking Stns 2W + 3w + SBI	–	–	0.16 (141,5?57)	0.15 (S5,6S2)		- -
STS Data	–	–	0.03 (23,333)	–	0.02	0.01

* Includes: 3 DSN Stations 2w+3w; 2 Russian Stations 3w; 1 German Station 3w.
+ Entries within parentheses indicate the total number of data points for the case.

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